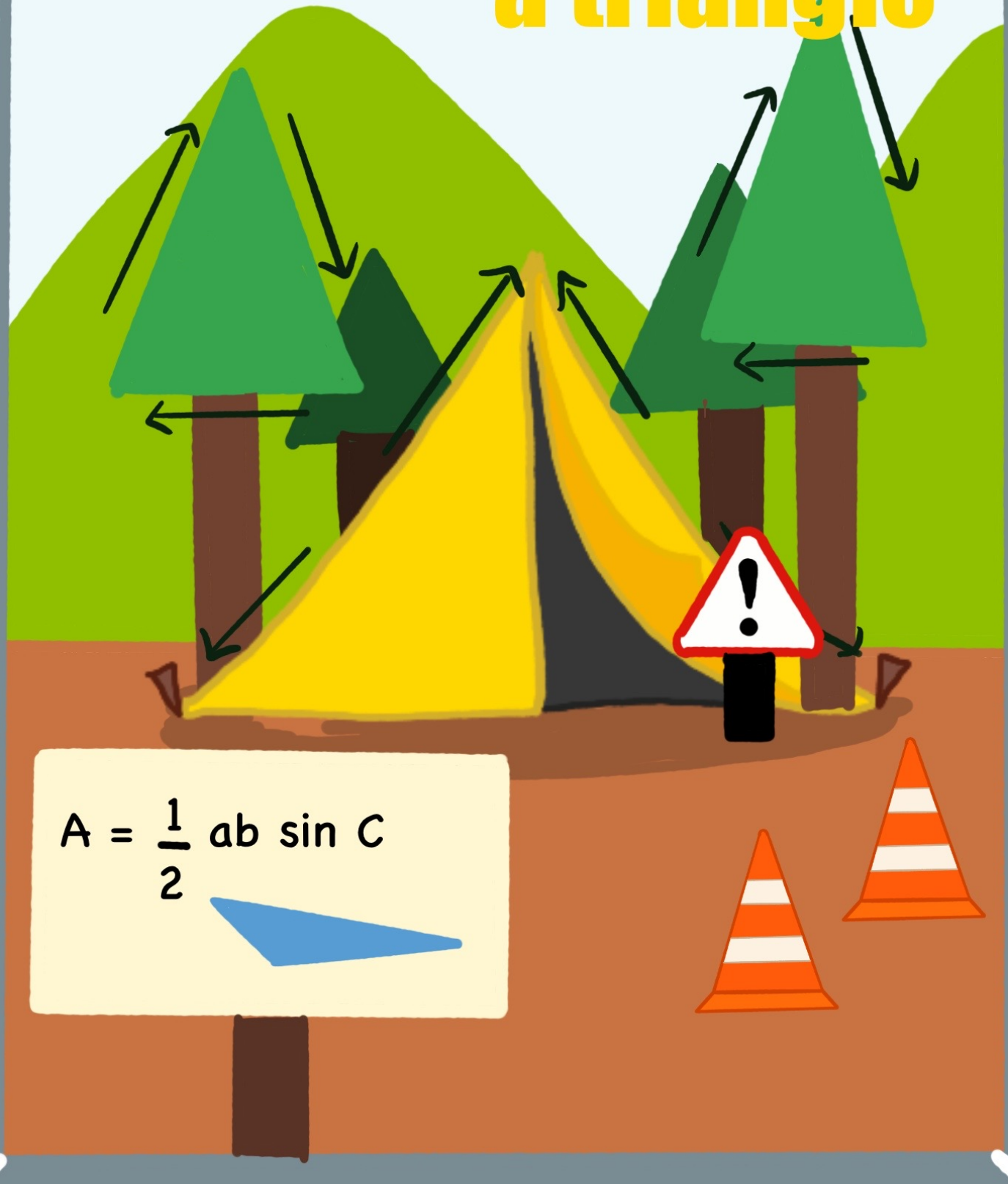
The Maths Society

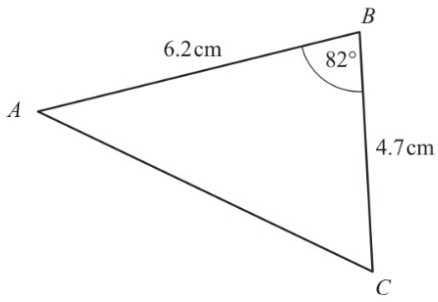
Area of a triangle



$$A = \frac{1}{2}\,ab\,\sin C$$

Question 1

(a)



NOT TO SCALE

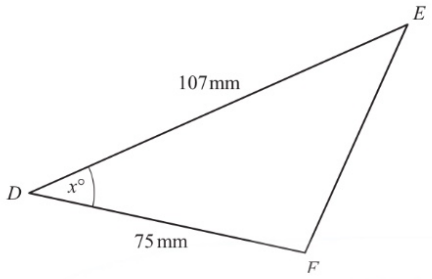
Calculate the area of triangle ABC. [2]

$A = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 6.2 \times 4.7 \times \sin 82°$

$= 14.4\,cm^2$

(b)



NOT TO SCALE

The area of triangle DEF is $2050mm^2$.

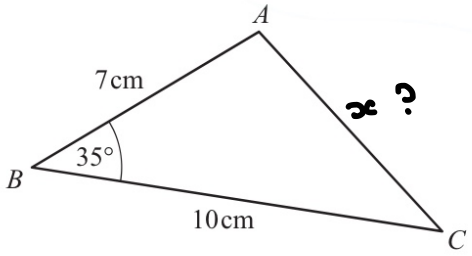
Work out the value of x. [2]

$A = \frac{1}{2}ab\sin C$

$2050 = \frac{1}{2} \times 107 \times 75 \times \sin x$

$\sin x = \frac{164}{321}$ $\quad x = \sin^{-1}\left(\frac{164}{321}\right) = 30.7°$

Question 2



NOT TO SCALE

(a) Calculate the area of triangle ABC. [2]

$A = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 7 \times 10 \times \sin 35°$

$= 20.1\,cm^2$

(b) Calculate the length of AC. [4]

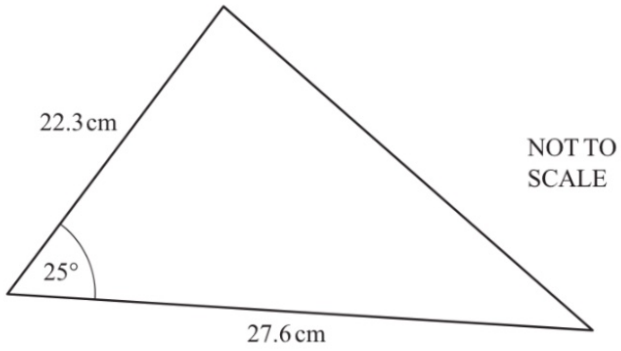
$\cos\theta = \frac{b^2 + c^2 - x^2}{2bc}$

$\cos 35 = \frac{7^2 + 10^2 - x^2}{2(7)(10)}$

$\cos 35 \times 140 = 49 + 100 - x^2$

$x^2 = 34.3$ $\quad x = 5.86cm$

Question 3



Calculate the area of this triangle.

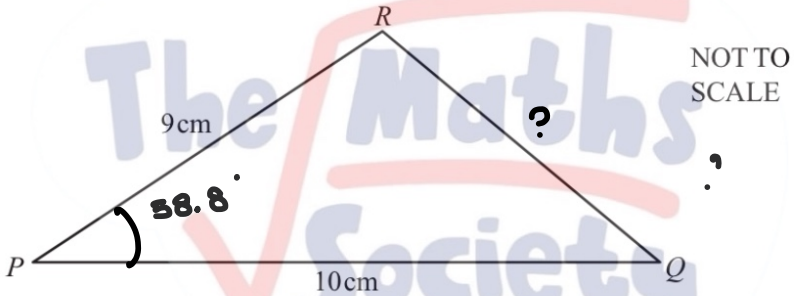
$A = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 22.3 \times 27.6 \times \sin 25°$

$= 130.1\text{cm}^2$

[2]

Question 4



The area of triangle PQR is 38.5cm^2.

Calculate the length QR.

$A = \frac{1}{2}ab\sin C$

$38.5 = \frac{1}{2} \times 9 \times 10 \times \sin \hat{P}$

$\sin \hat{P} = \frac{77}{90}$

$\hat{P} = \sin^{-1}\left(\frac{77}{90}\right)$

$= 58.8°$

$\cos \theta = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos 58.5 = \frac{9^2 + 10^2 - QR^2}{2(9)(10)}$

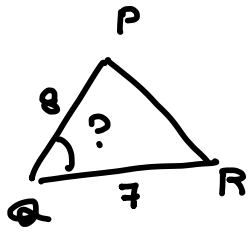
$\cos 58.5 \times 180 = 9^2 + 10^2 - QR^2$

$QR^2 = 86.950$

$QR = 9.3\text{ cm}$

[6]

Question 1



In a triangle PQR, $PQ = 8$ cm and $QR = 7$ cm.
The area of this triangle is 17 cm^2.

Calculate the two possible values of angle PQR. [3]

$A = \frac{1}{2}ab \sin C$

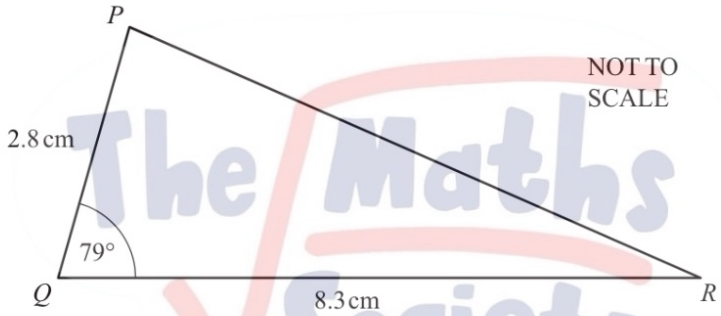
$17 = \frac{1}{2} \times 8 \times 7 \times \sin \hat{Q}$

$\sin \hat{Q} = \frac{17}{28}$

$\hat{Q} = 37.4°, 180° - 37.4°$

$= 37.4°, 142.6°$

Question 2



(a) Calculate the area of triangle PQR. [2]

$A = \frac{1}{2}ab \sin C$

$= \frac{1}{2} \times 2.8 \times 8.3 \times \sin 79$

$= 11.4 \text{ cm}^2$

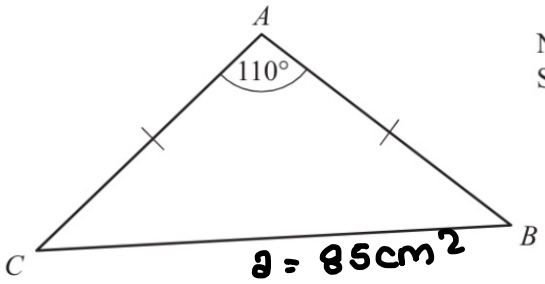
(b) Triangle PQR is enlarged by scale factor 4.5.

Calculate the area of the enlarged triangle. [2]

area $= 11.4 \times 4.5^2$

$= 231 \text{ cm}^2$

Question 3



NOT TO SCALE

Triangle ABC is isosceles with $AB = AC$.
Angle $BAC = 110°$ and the area of the triangle is $85\,cm^2$.

Calculate AC. [3]

$A = \frac{1}{2}ab\sin C$

$85 = \frac{1}{2} \times x \times x \times \sin 110$

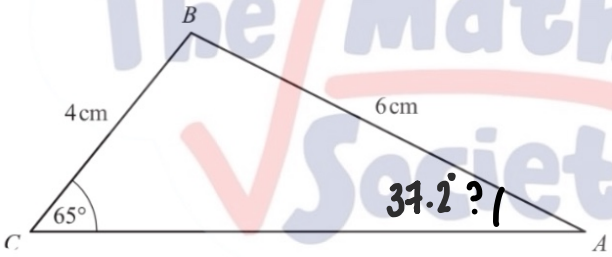
$\frac{85}{\sin 110} = \frac{1}{2}x^2$

$x^2 = 180.91$

$x = \sqrt{180.91}$

$AC = 13.4cm$

Question 4



NOT TO SCALE

In triangle ABC, $AB = 6\,cm$, $BC = 4\,cm$ and angle $BCA = 65°$.

Calculate

(a) angle CAB, [3]

$\frac{4}{\sin \hat{A}} = \frac{6}{\sin 65}$

$\sin \hat{A} = \frac{4 \times \sin 65}{6}$

$\hat{A} = \sin^{-1}\left(\frac{4 \times \sin 65}{6}\right)$

$= 37.2°$

(b) the area of triangle ABC. [3]

$\hat{B} = 180° - 65° - 37.2°$

$= 77.8°$

$A = \frac{1}{2} \times ab \sin C$

$= \frac{1}{2} \times 4 \times 6 \times \sin 77.8$

$= 11.7\,cm^2$